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TIME CRITERIA OF EXPLOSIVE FRACTURE

Yu. I. Fadeenko

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The total fracture of a solid in a given section presupposes the satisfaction of the following time criteria: 1) the fracture preparation criterion (damage accumulation, formation of embryonic cracks); 2) the integral crack coalescence criterion, based on the nonstationary crack growth equation.

In solving specific problems it may prove convenient to consider separately the initial (essentially nonstationary) phase of acceleration of cracks initially at rest and the subsequent phase of quasistationary growth; in this case the second of the above-mentioned criteria breaks down into two separate time conditions. The starting relations may also include Griffith's criterion, i.e., a differential crack growth condition requiring that the energy-release rate be not less than the work-absorption rate. Generally speaking, Griffith's criterion should be obtained from the crack growth equation by equating the growth rate to zero.

Thus, the total fracture time τ can be represented as the sum of the fracture preparation time τ_1, the duration of the transient process τ_2, and the period of quasistationary growth leading to total coalescence of the cracks τ_3:

$$\tau = \tau_1 + \tau_2 + \tau_3. \tag{1}$$

In recent years the kinetic theory of fracture has gained wide acceptance. The fundamental principles of the kinetic theory have received extensive experimental confirmation; for alloys and polymers they have proved to be so general that deviations from them have been the subject of special investigation. However, the experiments on which the theory is based relate to the region of large rupture lives (10^{-3} sec and more). Until recently it was uncertain whether the kinetic theory could be applied on the interval of short rupture lives (10^{-6} sec or less) typical of explosive fracture. Here it is shown that the region of applicability of the kinetic theory, as usually formulated, is limited and that on the interval of short rupture lives it should be substantially modified.

The basic relation of the kinetic theory – the time fracture criterion determining the rupture life τ [see (1)] of a solid subjected to the action of a constant tensile stress σ – is usually written in the following form:

$$\tau = \tau_0 \exp \frac{u - \gamma\sigma}{kT}, \tag{2}$$

where k is Boltzmann's constant; T is temperature; τ_0 is the preexponential coefficient, which coincides in order of magnitude with the period of the thermal vibrations of the atoms (10^{-13}–10^{-12} sec); u is the activation energy (of the order of the atomic bond energy in the solid).

The factor γ is a characteristic of the actual processes preparatory to fracture that take place at the atomic level. It is usually assumed that γ characterizes the most dangerous of the structural defects – the microstress raisers; the quantity γ, which has the dimension of volume, can be interpreted as the product of the volume of the defect and the stress-concentration factor.

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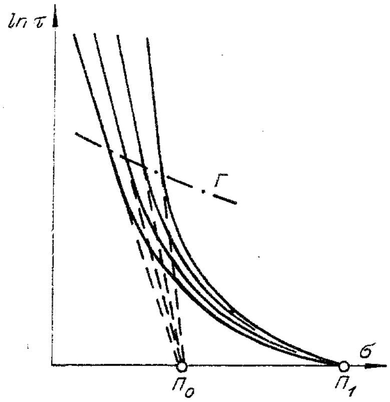

Fig. 1

So far the quantities u and $\gamma\sigma$, and hence the kinetic theory of fracture itself, have not received a completely acceptable physical interpretation. For example, it is possible to treat (2) as the result of combining the basic equation of the kinetic theory of flow with the fracture condition corresponding to the reaching of the critical plastic strain:

$$\dot{\varepsilon}^p = \text{const} \cdot \exp\left(-\frac{u-\gamma\sigma}{kT}\right), \quad \varepsilon^p \geqslant \varepsilon^p_*$$

(concerning the treatment of (2) see also [2]). However, the possible interpretations of (2) are not especially relevant to what follows.

Under ordinary conditions the quantity γ is equal to the volume of 20-500 atoms and for a particular material remains constant over a very broad range of values of τ. Accordingly, the family of isotherms T = const obtained from (2) in the plane (σ, ln τ) takes the form of a fan with rays converging at the point with coordinates (u/γ, ln τ_0) (the pole Π_0 in Fig. 1). Attempts to use (2) on the interval of small values of τ encounter chiefly the following difficulty: The abscissas of Π_0 do not coincide with theoretical strength σ_t, (u/γ) usually being several times smaller than σ_t. Clearly, the isotherms should converge at the true pole Π_1. For this it is necessary that at small τ the quantity γ, which characterizes the slope of the isotherms ($d \ln \tau/d\sigma = -\gamma/kT$), be a decreasing function of σ. Estimates show that as $\sigma \to \sigma_t$, γ should decrease to approximately the volume of a single atom. This implies that as $\sigma \to \sigma_t$ the rupture life of the interatomic bond becomes less dependent on the state of the surrounding atoms so that in the limit each bond breaks individually, s in an ideal crystal lattice.

It is characteristic of explosive fracture that at very high stresses and short loading pulses it is no longer possible to take $\tau \simeq \tau_1$ [as is implicitly assumed when (2) is used under ordinary conditions]. On the contrary, it may be that $\tau \simeq (\tau_2 + \tau_3) \gg \tau_1$. Moreover, it is important to consider the relations between these times and the length of the loading pulse.

The need to take τ_2 into account can be qualitatively explained as follows. Let Griffith's length be equal to l. For a potentially unstable (in Griffith's sense) crack to begin to grow, the conditions for elastic-wave transfer of the energy released to the edges of the crack must be established in the region around it. The time required for this can be estimated as $\tau_2 \simeq l/c$ (c is the speed of sound). Then for disklike cracks of radius 1 we have [3]

$$\sigma^2 = \pi\alpha E/2(1-\nu^2)l = \pi\alpha E/2(1-\nu^2)c\tau_2, \tag{3}$$

where ν is Poisson's ratio; α is the work done on forming unit area of the crack; E is Young's modulus. Equation (3) is the equation of a curve in the plane (σ, ln τ), represented schematically in Fig. 1 by the curve Γ. The value of α may vary over a range of several orders, depending on whether fracture is brittle or ductile; consequently, depending on the test conditions, the curve Γ may intersect the isotherm fan anywhere on a broad interval of τ, extending from subnanosecond to microsecond values.

If the loading time is long enough for the investigated process to correspond to points in Fig. 1 lying above the curve Γ, then fracture may begin, for example, as a result of several random adjacent microcracks coalescing into a single crack satisfying Griffith's criterion. Then during time $\tau_2 \simeq l/c$ energy is transferred to the edges of the crack, which acquires the ability to propagate through homogeneous material irrespective of the presence of other microcracks along its path, i.e., becomes a main fracture crack. The process ends with main cracks spreading over the entire cross section and the disintegration of the body into a small number of large fragments.

At points lying below the curve Γ time criterion (3) is known not to be satisfied for a homogeneous medium, i.e., for main cracks. Even if potentially unstable (in Griffith's sense) cracks are present, they cannot develop into growing main cracks. Accordingly, fracture is possible only if there are very many independent embryonic microcracks and they coalesce to cover the entire cross section. Since in this case there is no well-defined critical section, the process takes place uniformly throughout the material. It begins with the formation of a pore cloud, similar to a cloud of cavitation bubbles [4], and ends with the disintegration of the body. In this case, τ_1, determined from (2), represents the characteristic development time of the cavitation cloud. If, however, there are few cavitation pores and cavitation does not lead to the disintegration of the body, then fracture finally ensues when it becomes possible for main cracks to grow, i.e., when time criterion (3) is satisfied. In this case criterion (3) is the lower bound for the true time fracture criterion. As distinct from (2), this is an athermal criterion.

In the plane $(\sigma, \ln \tau)$ the slope of the Γ curves in the region where they intersect the isotherm fan is usually one or two orders less than that of the isotherms. Accordingly, there should be a sharp change in the character of the experimental $\tau(\sigma)$ curves in that region, as appears to have been observed in [5] and other similar studies.

The Bailey criterion [6]

$$\int \frac{dt}{\tau_1(\sigma)} = 1$$

extends relation (2) to the case of time-dependent processes.

In the particular case of the scabbing problem it is also possible to formulate a certain dynamic time criterion for the process, whose duration is determined not by (2) but by the crack coalescence time. Let part of the area S of a given section be occupied by cracks. The energy flux $c\sigma^2/E$, transported by an acoustic wave, impinges on the section. Part of the flux, proportional to $(1 - S)$, passes through the section, while the remainder is divided into a reflected flux, proportional to kS, and a fraction expended on developing the cracks, proportional to $(1 - k)S$ (k is the reflection coefficient, which, generally speaking, is assumed to be variable). The crack growth equation takes the form

$$(1 - k)(\sigma^2/E)cSdt = \alpha dS.$$

Integrating this equation, with allowance for the fact that during the fracture time S varies from a certain initial value S_0 to 1, we obtain

$$\int_0^{\tau_2} \frac{\sigma^2}{\alpha(t)} [1 - k(t)]\, dt = \frac{2E}{c} \ln \frac{1}{S_0},$$

which recalls the empirical criterion

$$\int (\sigma - \sigma_*)^n\, dt = I, \tag{4}$$

where σ_*, n, and I are parameters.

The time criteria considered are local fracture criteria. When the fracture conditions of some complete systems are analyzed, the basic starting relations include the fracture energy balance. This may be written, for example, in the following approximate form: The total energy of crack formation is equal to the sum of the potential and a certain fraction of the kinetic energy of the system at fracture. If the kinetic energy is neglected, the equation can be expressed as a relation formally analogous to Griffith's criterion (3) (attention was drawn to this possibility in [7, 8]), the difference being that the Griffith's length l is replaced by the characteristic dimension of the fragments. To avoid possible misunderstandings, it should be stressed that the energy balance equation for the system as a whole is not a fracture criterion, but makes it possible to determine the characteristic dimension (number) of the fragments formed if the fracture criterion is satisfied. If it is required to minimize the number of fragments, i.e., reduce them to two, the energy-balance equation can, of course, be regarded as a condition whose satisfaction is necessary (but not sufficient) for the failure of the system. However, estimates obtained using only this condition may considerably exaggerate the risk of failure.

These observations are not limited to the process of fracture at high stresses. They also apply whenever a body can pass from a metastable to a stable state under an external stimulus in either of two ways: as a result of a transition wave propagating from individual centers or as a result of a homogeneous volume transition, the times required for the centers to develop and the transition waves to be propagated throughout the body being comparable to the duration of action of the external source. In this sense dynamic fracture should be associated with a number of analogous effects.

An example is offered by the shock initiation of a condensed explosive, when τ is to be regarded as the total reaction time at the given shock temperature T and pressure p. Unfortunately, it is not possible to trace in detail the analogy between fracture and the initiation of an explosion, since the corresponding equations [the analogs of (2) and (4)] have so far been adequately studied only in relation to the temperature (but not pressure) dependence of τ. One of the few known results concerning the $\tau(p)$ relation is the equation

$$\tau = \text{const}/p^n, \tag{5}$$

proposed on the basis of experimental data on the reation time in TNT in the detonation and predetonation regimes; for loose and compressed TNT $n=1$ [9] and for cast TNT $n \geq 2$ [10]. In [11, 12] the "critical energy" concept is described. According to this concept hot spots develop in the initiated explosive when the condition $(p^2\tau/U) = \text{const}$ (U is the shock-wave velocity) is satisfied for the initiating shock. This condition is similar in form to (5). It is worth noting the analogy between Griffith's criterion in form (3) and Eq. (5) at the particular value $n=2$, typical of a mechanically strong explosive. However, it is not clear whether Eq. (5) is the rate equation of a reaction proceeding uniformly throughout the compressed volume of the explosive or a criterion of the formation of centers of explosion which, in a mechanically strong explosive, could be formed, for example, by a network of growing shear cracks.

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